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The Rationality Bias*

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Abstract

We analyze differences in consumption and wealth that arise because of different degrees of rationality of households. In particular, we use a standard New Keynesian model and let a certain fraction of households be fully rational while the other fraction possesses less cognitive ability. We identify the rationality bias of boundedly rational agents, defined as a deviation from the fully rational benchmark, as the driver of consumption and wealth heterogeneity. It turns out that the rationality bias can be decomposed into three individual components: the consumption expectation bias, the real interest rate bias and the preference shock expectation bias. We show that for certain specifications of monetary policy the rationality bias can be eliminated because its individual components exactly offset each other although they are individually non-zero. However, it might not be desirable from a welfare perspective to eliminate the rationality bias as this comes along with high inflation volatility.

Keywords: heterogeneous expectations, bounded rationality, consumption and wealth heterogeneity, monetary policy

JEL classifications: E32, E52, D84

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1 Introduction

Since the collapse of the global financial system in 2007-08 there is a growing consensus that models with homogeneous rational expectations cannot adequately approximate actual human behavior at the microeconomic or macroeconomic level. Even though there may be some highly sophisticated and well informed economic actors, a non-negligible fraction of the population might not be nearly as rational as assumed in theoretical models. In response to a growing need for models that take account of cognitive imperfections different strands of literature that incorporate bounded rationality have gained popularity. Notable examples are the adaptive learning literature (see, e.g. Evans and Honkapohja, 2012), sparse dynamic programming (Gabaix, 2016) and a strand of literature that incorporates heterogeneous, possibly non-rational expectations in macro models (Branch and McGough, 2016). However, papers in the heterogeneous expectations literature have focused on aggregate dynamics, largely ignoring the individual level. We want to fill this gap in the literature and explicitly consider how differences in cognitive ability lead to differences in consumption and wealth.

For this purpose, we set up a micro-founded model where a given fraction of agents is *fully rational* in the conventional sense (Muth, 1961), while the other fraction is *boundedly rational* as we will explain in detail below, and where we keep track of the individual bond holdings of both groups. We identify the rationality bias of boundedly rational agents, defined as a deviation from the fully rational benchmark, as the driver of consumption and wealth heterogeneity. Moreover, we find a strong interaction between the size of the rationality bias and monetary policy.

Empirical evidence based on US inflation survey data suggests that in the real world expectations are indeed heterogeneous and that a sizable fraction of the population follows simple backward-looking heuristics. For instance, Branch (2004) finds evidence of the presence of different expectation types in the Michigan Survey of Consumer Attitudes and Behavior. In particular, both naive expectations, where the last observed value of a variable is used as a best guess for the future, and a more sophisticated VAR heuristic are present in the data. Cornea-Madeira et al. (2017) further find that a model with naive agents and agents that use a VAR approach to predict inflation based on its forward-looking relation with marginal costs fits actual inflation data well. Moreover, simple backward-looking heuristics are consistent with evidence from laboratory experiments (Assenza et al., 2014; Pfajfar and Žakelj, 2016), where a large degree of heterogeneity is found. Also, Fuhrer (2017) identifies slow-moving expectations as a source of macroeconomic persistence by incorporating survey expectations into an otherwise standard DSGE model that makes ad-hoc assumptions like indexation and habit-formation to generate observed persistence obsolete.

In our model, boundedly rational agents form their expectations in a naive manner, consistent with Branch (2004) and Cornea-Madeira et al. (2017) and similar to earlier literature on heterogeneous expectations (Branch and McGough, 2009, 2010; De Grauwe, 2011; Gasteiger, 2014; Hommes and Lustenhouwer, 2019). Naive expectations perform well when the variable being forecasted is highly persistent and are optimal when the variable follows a random walk. Moreover, our boundedly agents base their consumption decision only on the variational intuition of the consumption Euler equation following Branch and McGough (2009). Boundedly rational agents hence do not make decisions according to the infinite horizon learning approach of Preston (2005) and Massaro (2013) which would require them to form expectations about all variables over an infinite horizon and make optimal decisions based on these expectations. We believe that this would require too much cognitive ability of boundedly rational agents.

The other fraction of agents, on the other hand, is fully rational in the conventional sense. These agents are fully aware of the presence of boundedly rational agents and choose the optimal intertemporal allocation of consumption, labor supply and bonds based on all their optimality conditions including their intertemporal budget constraint. Thus, we combine the Euler learning approach of Branch and McGough (2009) and Honkapohja et al. (2012) for boundedly rational agents with the infinite horizon learning approach of Massaro (2013) and Preston (2005) for rational agents. To the best of our knowledge, this is the first paper to do so.

In this framework we find that considerable consumption and wealth heterogeneity arises as the economy is hit by shocks because boundedly rational agents respond differently with their consumption decision than rational agents. The reason is that boundedly rational agents are subject to the rationality bias that we identify as a deviation from the fully rational benchmark. We find that the rationality bias can be decomposed into three different components: the consumption expectation bias, the real interest rate bias and the preference shock expectation bias.

Each of these biases lead bounded rational agents to deviate from the rational consumption decision. However, they may also counteract each other. For instance, when the economy is hit by a cost-push shock, and inflation falls after its initial hike, boundedly rational agents consistently overestimate inflation due to their naive forecast heuristic. This leads their subjective real interest rate to be lower compared to the objective one which puts upward pressure on their consumption decision. At the same time, a recession induced by the central bank (to bring down inflation) leads them to overestimate the recession, now putting downward pressure on their consumption decision.

Our main finding is that the rationality bias disappears when the percentage deviation from steady state of the nominal interest rate and the inflation rate is equal in every period and when the economy was initialized without consumption and wealth heterogeneity (as in steady state). In this case, the three components of the rationality biases exactly offset each other no matter what the realizations of shocks are. We further show that under cost-push shocks the rationality bias can be eliminated with any response of the nominal interest rate to inflation that is larger than one and a corresponding positive response to output. Under preference shocks, on the other hand, this is only possible if it is assumed that the central bank can observe and respond to the shocks contemporaneously. However, it might not be desirable from a welfare perspective to eliminate the rationality bias as it comes along with high inflation volatility. Even though consumption heterogeneity enters the utilitarian welfare loss of this model, the weight on inflation suggest that the households populating this economy prefer stable prices over homogeneous consumption (and output stability) by far.

As previously indicated, the literature on heterogeneous expectations (see for instance Branch and McGough (2016)) mainly focuses on aggregate dynamics and does not explicitly consider differences in consumption and wealth of the different agent types. Recent exceptions are Beqiraj et al. (2017) and Annicchiarico et al. (2018). The former calculate a measure of consumption inequality in their economy. The latter present differences in bond holdings that arise between some of their agents. The main interest of these studies, however, remains aggregate dynamics. To the best of our knowledge, there is no paper in this tradition that *focuses* on individual agent's behavior in a macro-setting.

The remainder of the paper is organized as follows. The model is introduced in Section 2 where we put emphasis on the assumptions on bounded rationality. In Section 3, we show how the rationality bias can be decomposed into three specific cognitive biases of boundedly rational agents and how the economy evolves dynamically in response to a cost-push and preference shock. Finally, in Section 4 we show that the rationality bias can be eliminated with certain monetary policy and include a short remark on welfare. Section 5 concludes.

2 Model

In this Section, we introduce heterogeneous rationality into a standard New Keynesian model. In particular, we assume a unit-mass continuum of households $i \in [0, 1]$ and firms $j \in [0, 1]$ as well as a monetary and fiscal authority. The population of households splits into two groups with a fixed size: rational households, who make up a fraction α of the population and boundedly rational households, making up the remaining $1 - \alpha$ of the population.

There is an ongoing discussion in the literature (see e.g. Honkapohja et al., 2012) whether boundedly rational agents should be assumed to base their consumption decision only on the variational intuition of the consumption Euler equation (Euler equation learning) or instead make optimal decisions based on boundedly rational forecasts of the entire future paths of all variables (infinite horizon learning). Since we study differences in consumption and wealth that arise due to heterogeneity in cognitive ability, we let our two types of agents differ considerably in their degrees of rationality. For this reason, we stick with the less cognitive demanding Euler equation learning approach for our boundedly rational agents. We, however, do not follow Branch and McGough (2009) and the related literature on heterogeneous expectations by letting agents with rational expectations also use Euler equation learning. Instead, we combine the Euler equation learning of boundedly rational agents with fully rational optimization in the conventional sense of rational agents. Detailed descriptions of both household types including their individual expectation formation schemes are included in Sections 2.2 and 2.3.

Further, we assume perfect consumption insurance within the groups, so that the two types of agents can be interpreted as two different representative agents. Firms are assumed to be run be rational managers only, as boundedly rational managers will not survive in a competitive evolutionary process.

2.1 The non-linear model

First, we introduce the basic model elements in non-linear form. The log-linearized version including a specification of each household type and policy rules is given in the subsequent section.

2.1.1 Households first-order conditions

Households of type i with $i \in \{R, B\}$ optimize their expected lifetime utility $E_t^i \sum_{t=0}^{\infty} \beta^t \Upsilon_t U_t^i$ subject to their real flow budget constraint

$$C_t^i + \bar{Y} b_t^i \leq W_t H_t^i + \bar{Y} \frac{I_{t-1}}{\Pi_t} b_{t-1}^i + D_t - T_t \tag{1}$$

where E_t^i is the type-specific expectations operator, $\beta < 1$ the subjective discount factor, U_t^i period-utility of type i and Υ_t is a preference shock. C_t^i denotes individual consumption, $b_t^i = \frac{B_t^i}{P_t \bar{Y}}$ individual real debt-to-steady-state-output where $\bar{Y}$ is the steady state of output Y_t, T_t is a lump-sum tax and W_t the real wage rate, which are equal across groups. Further, H_t^i denotes individual hours, I_{t-1} the gross nominal interest rate controlled by the central bank in period $t-1$, $\Pi_t = \frac{P_t}{P_{t-1}}$ the inflation rate where P_t is the aggregate price level in t and D_t dividends that households obtain as shareholders of the firms.

Period utility is of CES-form and given by

$$U_t^i = \frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{(H_t^i)^{1+\gamma}}{1+\gamma} \tag{2}$$

where σ is the coefficient of relative risk aversion and γ the labor supply elasticity.

Households first-order conditions are (1) and

$$\Upsilon_t (C_t^i)^{-\sigma} = \beta E_t^i \Upsilon_{t+1} (C_{t+1}^i)^{-\sigma} \frac{I_t}{\Pi_{t+1}} \tag{3}$$

$$W_t = (H_t^i)^{\gamma} (C_t^i)^{\sigma} \tag{4}$$

2.1.2 Firms

We assume that firms are run by rational managers that are appointed by households (who are the shareholders of firms). In the end, this assumption allows us to isolate the effect of *heterogeneous rationality of households* on differences in consumption and wealth. The assumption that all firm managers are rational can be justified as follows: one could imagine an evolutionary process where rational managers will outperform boundedly rational ones, and hence are the only surviving managers of firms. That is, if we would explicitly model appointment and firing of managers and allow for bankruptcy of firms, then a firm that is led by a boundedly rational manager, who consistently performs worse than its fully rational competitors, would either get fired or lead its firm into bankruptcy. Thus, at any point in time, the fraction of boundedly rational managers would be close to zero.

Production is linear in labor, while we abstract from variations in total factor productivity which is constant over time and equal to one. Thus,

$$Y_t(j) = H_t(j) \tag{5}$$

where $H_t(j)$ is labor supply for good j.

We assume Calvo pricing where a firm j is able to re-set its price only with a given probability of $1-\omega$ in each period. Thus, each firm j maximizes its current and future discounted flow of profits by setting its price $P_t(j)^*$ subject to the demand of differentiated goods $Y_t(j)$ by the final goods producer according to

$$\max_{P_t(j)^*} \quad E_t \sum_{s=0}^{\infty} \omega^s Q_{t|t+s} [P_t(j)^* Y_{t+s}(j) - Y_{t+s}(j) P_{t+s} MC_{t+s}] \tag{6}$$

$$s.t.$$

$$Y_{t+s}(j) = \left(\frac{P_t(j)^*}{P_{t+s}}\right)^{-\eta} Y_{t+s} \tag{7}$$

where $Q^s_{t|t+s} = \beta^s \frac{\Upsilon_{t+s}}{\Upsilon_t} \left(\frac{C_{t+s}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+s}}$ is the stochastic discount factor with C_t being aggregate consumption.

The optimization procedure gives firm j's first-order condition

$$\tilde{P}_t(j)E_t\sum_{s=0}^{\infty}\omega^s\beta^s\Upsilon_{t+s}C_{t+s}^{-\sigma}\left(\frac{P_{t+s}}{P_t}\right)^{\eta-1}Y_{t+s}=\frac{\eta}{\eta-1}E_t\sum_{s=0}^{\infty}\omega^s\beta^s\Upsilon_{t+s}C_{t+s}^{-\sigma}\left(\frac{P_{t+s}}{P_t}\right)^{\eta}Y_{t+s}MC_{t+s}. \quad (8)$$

where $\tilde{P}_t(j)=\frac{P_t(j)^*}{P_t}$.

2.1.3 Government and market clearing

The real government budget constraint reads

$$b_t=g_t-\frac{T_t}{\bar{Y}}+s\frac{H_t}{\bar{Y}}W_t+\frac{I_{t-1}}{\Pi_t}b_{t-1} \quad (9)$$

where $g_t=\frac{G_t}{\bar{Y}}$ is government spending relative to steady state output and $s=\frac{1}{\eta}$ is a subsidy provided to firms (as in Galí, 2008) that makes the steady state of our model efficient. We provide the description of the evolution of government spending, taxes and the nominal interest rate in the log-linear version of the economy.

Goods and bond markets clear according to

$$Y_t=(\alpha C_t^R+(1-\alpha)C_t^B)+\bar{Y}g_t \quad (10)$$

$$b_t=\alpha b_t^R+(1-\alpha)b_t^B. \quad (11)$$

2.2 Expectations

As already indicated, our two types of agents employ different forecasting rules. Rational agents are sophisticated enough to compute an optimal linear forecast, i.e. they use conditional statistical expectations $E_t^R x_{t+1}=E_t x_{t+1}$, while boundedly rational agents use the last observed value (x_{t-1}) of a variable as their best guess for the future with the following naive heuristic $E_t^B x_{t+1}=x_{t-1}$.

2.3 The log-linear economy

For our policy analysis, we use a log-linear version the model that is approximated around a non-stochastic zero-inflation steady state.

2.3.1 Individual consumption, labor and bonds

Households first-order conditions in log-linearized terms are

$$c_t^i = E_t^i c_{t+1}^i - \frac{1}{\sigma}(i_t - E_t^i \pi_{t+1} - \upsilon_t + E_t^i \upsilon_{t+1}) \tag{12}$$

$$w_t = \gamma h_t^i + \sigma c_t^i \tag{13}$$

$$\hat{b}_t^i = h_t^i + w_t + \beta^{-1}\hat{b}_{t-1}^i + \bar{b}\beta^{-1}(i_{t-1} - \pi_t) + \eta^{-1}d_t - (1-\bar{g})c_t^i - \frac{\bar{T}}{\bar{Y}}\tau_t \tag{14}$$

where lower-case letter indicate a log-deviation from steady state (and τ_t indicates log-deviation of taxes from steady state). Further, we denote $\hat{b}_t^i = b_t^i - \bar{b}$, and use $\bar{D} = \bar{Y} - (1-s)\bar{W}\bar{H} = \eta^{-1}\bar{Y}$ where $\bar{W} = 1$. All steady state values are derived in Appendix A.5.

Equation (14) implies that the budget constraint of rational agents is given by

$$\hat{b}_t^R = h_t^R + w_t + \beta^{-1}\hat{b}_{t-1}^R + \bar{b}\beta^{-1}(i_{t-1} - \pi_t) + \eta^{-1}d_t - (1-\bar{g})c_t^R - \frac{\bar{T}}{\bar{Y}}\tau_t. \tag{15}$$

Iterating (15) until infinity, substituting for all choice variables and plugging in expectations, $E_t^R = E_t$, gives the consumption decision of rational agents:

$$\begin{aligned} c_t^R = {} & \zeta\hat{b}_{t-1}^R + \zeta\bar{b}(i_{t-1} - \pi_t) + \zeta\beta E_t \sum_{s=t}^{\infty} \beta^{s-t}[\Gamma_y y_s - \Gamma_g \hat{g}_s - \Gamma_\tau \tau_s] \\ & - \frac{(1-\zeta\bar{b}\sigma)\beta}{\sigma} E_t \sum_{s=t}^{\infty} \beta^{s-t}(i_s - \pi_{s+1}) + \frac{\beta}{\sigma} E_t \sum_{s=t}^{\infty} \beta^{s-t}(\upsilon_s - \upsilon_{s+1}) \end{aligned} \tag{16}$$

where composite parameters ζ, Γ_y, Γ_g, Γ_τ and the derivation of (16) are given in Appendix A.1. Thus, fully rational households make predictions over the *entire paths* of output, government spending, taxes, real interest rates and preference shocks *until infinity*.

For boundedly rational agents, we consider such an approach, where they need to form expectations about the entire future paths of all variables and make optimal decisions based on these variables, to require too much cognitive load. Therefore, our boundedly rational agents follow Euler-equation learning and believe that all other agents will form the same beliefs as they do (higher-order beliefs assumption) as in Branch and McGough (2009). This implies that boundedly rational agents will neglect their intertemporal budget constraint as an *optimality condition.*

Boundedly rational agents are furthermore assumed to know that market clearing holds and that rational agents will also satisfy their consumption Euler equation. In addition to this, we do not require boundedly rational agents to be able to iterate their consumption Euler equation until infinity but only until an arbitrary period N which they consider to be the "long-run".

In Appendix A.2 it is shown that under the above assumptions the consumption decision of boundedly rational agents is given by

$$c_t^B = \frac{1}{1-\bar{g}} E_t^B y_{t+1} + E_t^B (c_N^B - c_N) - \frac{1}{\sigma}[i_t - E_t^B \pi_{t+1} - \upsilon_t + E_t^B \upsilon_{t+1}] - \frac{1}{1-\bar{g}} E_t^B \hat{g}_{t+1}. \tag{17}$$

Further, we assume that when boundedly rational agents have more current (beginning-of-period) wealth than the average, they will expect to able to consume more than the average in the "long-run" N, i.e. $E_t^B (c_N^B - c_N) = \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1})$. This assumption ensures that we can rule out equilibria in which the bond holdings of individual households are explosive and where households are able to roll over their debt until infinity, similar to including bonds in the utility function as in Kurz et al. (2013) and to quadratic adjustment costs of bonds in the budget constraint as proposed by Schmitt-Grohé and Uribe (2003). Also, this assumption re-introduces the budget constraint to the decision making procedure of boundedly rational agents but in a rather behavioral manner. We choose the parameter ψ to be high enough to rule out explosiveness but also low enough to have a small impact on short-run dynamics.

Plugging in expectations, $E_t^B x_{t+1} = x_{t-1}$, and the assumption about "long-run" beliefs gives

$$c_t^B = \frac{1}{1-\bar{g}} y_{t-1} + \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1}) - \frac{1}{\sigma}[i_t - \pi_{t-1} - \upsilon_t + \upsilon_{t-1}] - \frac{1}{1-\bar{g}} \hat{g}_{t-1}. \tag{18}$$

Comparing (18) with (16) shows that we impose far less cognitive load on boundedly rational agents than on rational ones as discussed above. While boundedly rational agents are forward looking in the sense that they forecast one-period-ahead (and the "long-run"), they become ultimately backward-looking due to their behavioral heuristics.

Bond holdings of rational and boundedly rational agents are given by (14) where we substitute for hours and wages which gives

$$\hat{b}_t^i = \beta^{-1}\hat{b}_{t-1}^i - \Gamma_c c_t^i + \Gamma_y y_t + \bar{b}\beta^{-1}(i_{t-1} - \pi_t) - \Gamma_g \hat{g}_t - \Gamma_\tau \tau_t \tag{19}$$

where the composite parameter Γ_c is given in the Appendix A.1.

2.3.2 Aggregate equations

Inserting (16) and (18) into the log-linearized goods market clearing equation, $y_t = (1-\bar{g})(\alpha c_t^R + (1-\alpha)c_t^B) + \hat{g}_t$, and writing this equation recursively yields aggregate output:

$$\begin{aligned} y_t = {} & \Phi_1 E_t y_{t+1} + \Phi_2 y_{t-1} + \Phi_3 E_t \pi_{t+1} - \Phi_4 \pi_t + \Phi_5 \pi_{t-1} + \Phi_6 i_{t-1} - \Phi_7 i_t \\ & + \Phi_8 E_t i_{t+1} - \Phi_9 \hat{b}_{t-1}^R + \Phi_{10} b_t^R - \Phi_{11} \hat{g}_{t-1} + \Phi_{12} \hat{g}_t - \Phi_{13} E_t \hat{g}_{t+1} \\ & - \Phi_{14} \tau_t + \Phi_{15} \hat{b}_{t-1} - \Phi_{16} \hat{b}_t + \Phi_{17} \upsilon_t - \Phi_{18} E_t \upsilon_{t+1} - \Phi_5 \upsilon_{t-1} \end{aligned} \tag{20}$$

where coefficients and the derivation of (20) are given in the Appendix A.3.

Inflation is standard and follows a forward-looking Phillips-curve under rational expectations. As shown in Appendix A.4 this implies,

$$\pi_t = \delta(\gamma + \frac{\sigma}{1-\bar{g}})y_t - \delta\sigma(1-\bar{g})^{-1}\hat{g}_t + \beta E_t[\pi_{t+1}] + \mu_t \tag{21}$$

with $\delta = \frac{(1-\omega\beta)(1-\omega)}{\omega}$ and μ_t a cost-push shock.

The log-linearized government budget constraint is given by

$$\hat{b}_t = \hat{g}_t - \Gamma_\tau \tau_t + s(w_t + h_t) + \bar{b}\beta^{-1}(i_{t-1} - \pi_t) + \beta^{-1}\hat{b}_{t-1} \tag{22}$$

We assume that government spending remains at its steady state level ($\hat{g}_t = 0$). Further, the costs of the subsidy, $s(w_t + h_t)$, are directly financed by lump-sum taxes. Additionally, taxes are assumed to respond to beginning-of-period debt. Lump-sum taxes therefore evolve as

$$\tau_t = \phi_{b,\tau}\hat{b}_{t-1} + \frac{s}{\Gamma_\tau}\left(1 + \gamma + \frac{\sigma}{1-\bar{g}}\right) y_t. \tag{23}$$

where we used $h_t = y_t$ and $w_t = \left(\gamma + \frac{\sigma}{1-\bar{g}}\right) y_t$ (since $\hat{g}_t = 0$).

The evolution of aggregate debt can therefore be written as

$$\hat{b}_t = \left(\beta^{-1} - \Gamma_\tau \phi_{b,\tau}\right) \hat{b}_{t-1} + \bar{b}\beta^{-1}(i_{t-1} - \pi_t). \tag{24}$$

Moreover, log-linearizing the bond market clearing equation gives

$$\hat{b}_t = \alpha \hat{b}_t^R + (1-\alpha)\hat{b}_t^B. \tag{25}$$

For central bank policy we assume a standard Taylor rule based on contemporaneous output and inflation, i.e.

$$i_t = \phi_\pi \pi_t + \phi_y y_t. \tag{26}$$

The preference shock and cost push-shock in the linearized model are assumed to follow AR(1) processes:

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \epsilon_\upsilon \tag{27}$$

$$\mu_t = \rho_\mu \mu_{t-1} + \epsilon_\mu \tag{28}$$

where ϵ_μ and ϵ_υ are uncorrelated IID shocks.

3 The Rationality Bias

In this section, we analyze differences in consumption and wealth that arise due to different degrees of rationality. Before turning to the question how monetary policy affects these differences in Section 4, we first define and then study in detail the rationality bias of boundedly rational agents.

3.1 The components of the rationality bias

Differences in wealth between boundedly rational and rational agents arise as a consequences of differences in their consumption. In this section we therefore focus on the latter. The difference in consumption arises because boundedly rational agents follow Euler-equation learning, believe that all other agents form the same expectations as they do and use a simple naive forecast heuristic to form these expectations. If a boundedly rational agent would not have these limitations it would act as a rational agent. Hence, the difference $\Delta_i c_t^i = c_t^B - c_t^R$ can be interpreted as the *bias of a boundedly rational agent*. We therefore label this difference the *Rationality bias*.

Using the individual consumption Euler equation, (12), of both agent types we can write

$$\Delta_i c_t^i = c_t^B - c_t^R = (E_t^B c_{t+1}^B - E_t c_{t+1}^R) - \frac{1}{\sigma}(rr_t^B - rr_t) - \frac{1}{\sigma}(E_t^B \upsilon_{t+1} - E_t \upsilon_{t+1}). \tag{29}$$

Equation (29) shows that we are able to decompose this bias into three individual sources: the consumption expectation bias, the real interest rate bias and the preference shock expectation bias.

In Section 2.3.1 we derived the consumption decision of boundedly rational agents (18) that can be rearranged as

$$c_t^B = \underbrace{\frac{1}{1-\bar{g}}(y_{t-1} - \hat{g}_{t-1}) + \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1})}_{=E_t^B c_{t+1}^B} - \frac{1}{\sigma}\underbrace{[i_t - \pi_{t-1}]}_{=rr_t^B} + \frac{1}{\sigma}[\upsilon_t - \underbrace{\upsilon_{t-1}}_{=E_t^B \upsilon t+1}]. \tag{30}$$

Comparing Equation (30) with the Euler equation of boundedly rational agents (12) shows explicitly what their consumption expectation and their subjective real interest rate is, i.e. $E_t^B c_{t+1}^B = \frac{1}{1-\bar{g}}(y_{t-1} - \hat{g}_{t-1}) + \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1})$ and $rr_t^B = i_t - \pi_{t-1}$. Inserting the boundedly rational consumption expectations, preference shock expectations and their subjective real interest rate into (29) and using that $\hat{g}_t = 0$ in all periods yields

$$\begin{aligned} \Delta_i c_t^i = c_t^B - c_t^R = &\left(\frac{1}{1-\bar{g}} y_{t-1} + \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1}) - E_t c_{t+1}^R\right) \\ &+ \frac{1}{\sigma}(\pi_{t-1} - E_t \pi_{t+1}) - \frac{1}{\sigma}(\upsilon_{t-1} - E_t \upsilon_{t+1}). \end{aligned} \tag{31}$$

The first term in (31) shows that the consumption expectation bias is largely driven by boundedly rational expectations about output (recall that we assume ψ to be a low number). Further, the real interest rate bias is merely an inflation expectation bias. Also, (31) shows that the rationality bias further includes the preference shock expectation bias (which is however not present under a cost-push shock).

In principle, the individual biases can amplify or counteract each other. In section 4 we will show that these biases exactly off-set each other when the percentage deviations from steady state of the inflation rate and the nominal interest rate are equal and when the economy was initialized without heterogeneity (as in steady state).

3.2 Baseline Calibration

In order to further study the rationality bias and the dynamics of aggregate variables, we parameterize the model as shown in Table 1. We emphasize, however, that our main results are derived analytically without parameterizing the model. We choose most of the parameter values for our baseline calibration to be in line with standard literature. As indicated in Section 2.3, we pick a small number for the responsiveness of boundedly rational consumption with respect to the difference of individual and aggregate wealth ψ, i.e. 0.05, as it is high enough to rule out explosive equilibria but also low enough to have a small impact on the short-run dynamics. We assume a fraction of 0.5 of rational and boundedly rational agents. Additionally, we assume AR(1)-processes for the cost-push and the demand shock. In both cases, the AR-parameter is assumed to be 0.85 while the i.i.d. component in the shock process is normally distributed with zero mean and a standard deviation of 0.025.

Micro parameter	$\beta = 0.99$	$\sigma = 2$	$\eta = 7.84$	$\gamma = 2$	$\omega = 0.75$
Expectations	$\alpha = 0.5$	$\psi = 0.05$			
Policy parameter	$\phi_y = 0.2$	$\phi_\pi = 1.5$	$\phi_{b,\tau} = 1$	$\bar{g} = 0.21$	$\bar{\tau} = 0.375$
Shock parameters	$\rho_\mu = 0.85$	$\rho_v = 0.85$	$sd(\epsilon_d) = 0.025$	$sd(\epsilon_v) = 0.025$	

Table 1: Baseline calibration

3.3 Model dynamics

With the parameterized model, we can use impulse responses to get more insight into the components of the rationality bias of boundedly rational agents for consumption and wealth heterogeneity and aggregate dynamics. Figure 1 and 2 depict the impulse responses following a persistent 2.5 percent cost-push and preference shock, *respectively*, under the baseline calibration.

In case of the cost-push shock, the preference shock expectation bias is zero. Thus, in Figure 1 we need to consider only two different components of the rationality bias: first, the real interest rate bias and, second, the consumption expectation bias.

The impulse responses in Figure 1 show that the cost-push shock raises inflation substantially more than one-for-one under both homogeneous rational expectations (dotted gray) and heterogeneous rationality (solid green). This is due to the persistence of the shock. Consequently, the central bank reacts by increasing the nominal interest rate in order to contract demand and to bring down inflation.[1]

However, rational and boundedly rational agents react quite differently to the increase in inflation and in the policy rate. The middle-right panel reveals the real interest rate bias which is driven by the inflation expectations of boundedly rational agents: while the rational agent's real interest rate (solid green) increases by less than three percent, the subjective real interest rate of boundedly rational agents (dashed red) increases by *six percent*. This is because boundedly rational agents base their expectations on the last period and have not observed the effects of the shock yet. Thus, they do not anticipate an increase in inflation which results in a quite extreme hike in their subjective real interest rate relative to the rational real rate in the initial period. However, from period 2 onward, where inflation is falling, boundedly rational agents will consistently overestimate future inflation which results in a subjective real interest rate that is persistently below the real rate of rational agents. Eventually, inflation expectations align when model variables approach the steady state which results also in an alignment of the subjective real interest rates.

These difference in subjective real interest rates partly explain the differences in consumption and bond holdings observed in the middle-left and middle panel of Figure 1. Rational agents, however, do not only recognize the relatively moderate increase in the real interest rate on impact but also that the real interest rate will stay high for some time, resulting in low future consumption and output. To fully explain the consumption and bond dynamics, we need to consider the

[1]Note that the slightly unusual pattern of lump-sum taxes in Figure 1 can be explained by the two components of the tax rule, (23). On the one hand, taxes respond positively to debt, but on the other hand, lump-sum taxes are used to finance the labor cost subsidy to firms. The costs of this subsidy (and hence taxes) go down as output falls. Lump-sum taxes, however, hardly affect the consumption of the two agent types (and hence output and inflation), since rational agents are Ricardian, and boundedly rational agents do not use their budget constraint as an optimality condition.

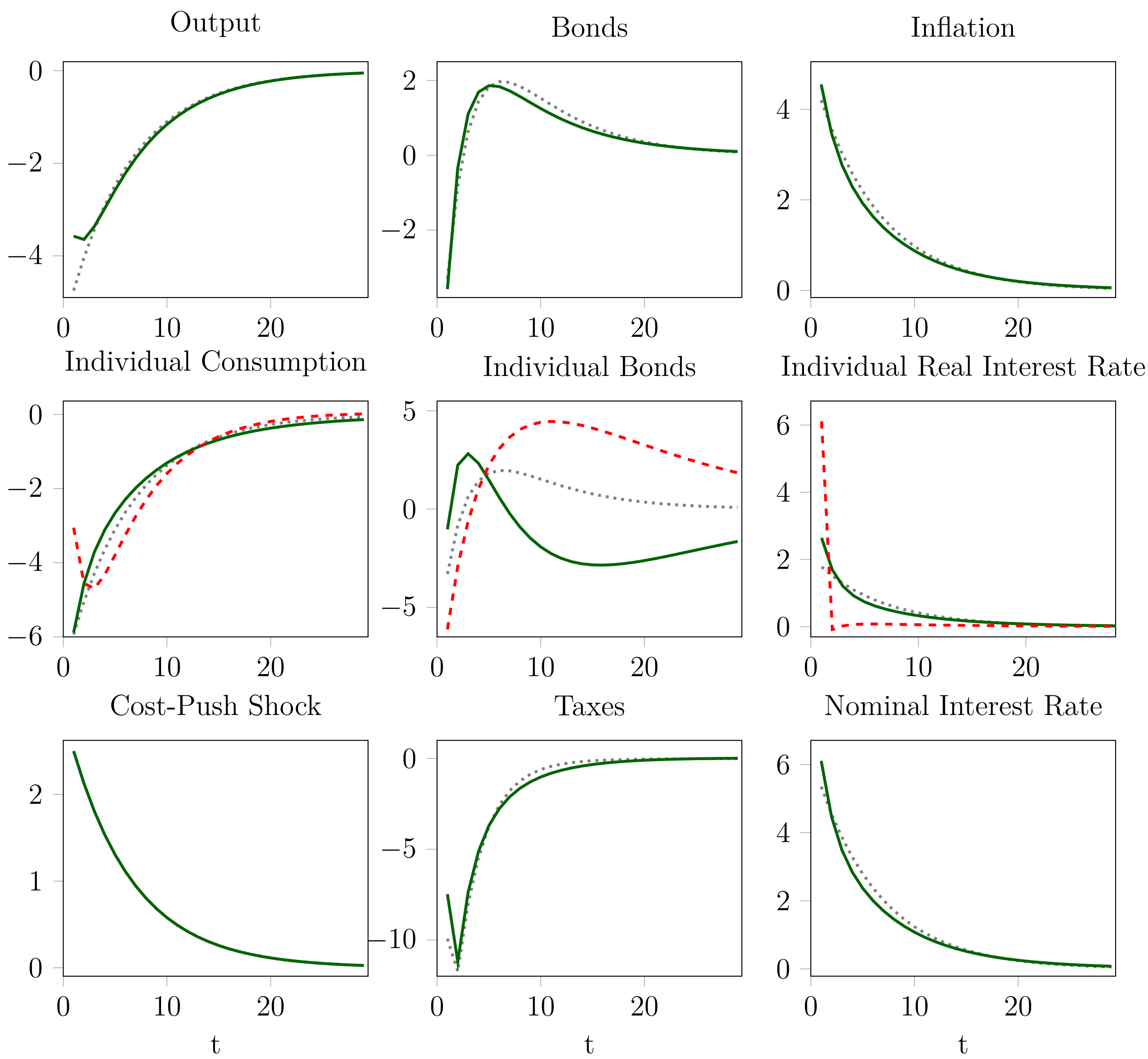

Figure 1: Impulse responses for a persistent 2.5% cost-push-shock under baseline calibration

consumption expectation bias of boundedly rational agents as well.

Boundedly rational agents do not anticipate a decrease in output and consumption in the shock period. Thus, their initial consumption response is solely due to the increase in their subjective real interest rate. However, in the next period, boundedly rational agents observe the recession caused by the shock and expect low future output, causing them to decrease their consumption further even though their subjective real interest is now considerably lower. The latter causes their consumption, however, to still be a little bit less negative than that of rational agents. This is no longer the case in later periods where output starts to pick up and output expectations of boundedly rational agents become too pessimistic which counteracts the positive effect of the real interest rate bias.

In the medium-run, boundedly rational agents are therefore consuming less than rational agents, and are accumulating more bonds. However, the higher consumption in the first periods caused boundedly rational agents to lose more bonds in the initial periods.[2] Therefore, wealth differences in the short-run arise because rational agents own more bonds, while in the medium-run boundedly rational agents are more wealthy. The latter allows boundedly rational agents to consume out of wealth in the medium-run so that their consumption can pick up again. Note, though, that since rational agents are fully optimizing using optimal linear forecasts, the consumption utility losses of boundedly rational agents over the life-cycle are higher than that of rational agents by definition.

The pattern of individual consumption is also reflected in the behavior of output that is not as low as under homogeneous rational expectations at the beginning due to the consumption decision of boundedly rational agents.

Next, we consider the impulse responses to a persistent preference shock in Figure 2. The shock itself shifts the preferences of both agent types such that they want to consume more and save less in the current period. However, as can be seen in Equation (12), agents' consumption does not only depend on today's shock realization but also on their expectations about future shock realizations. Rational agents expect a lower but still substantial shock realization tomorrow due to the shock persistence. Therefore, their consumption increase today will be substantially

[2]Even though households cut consumption, initially their real bond holdings go down due to high inflation.

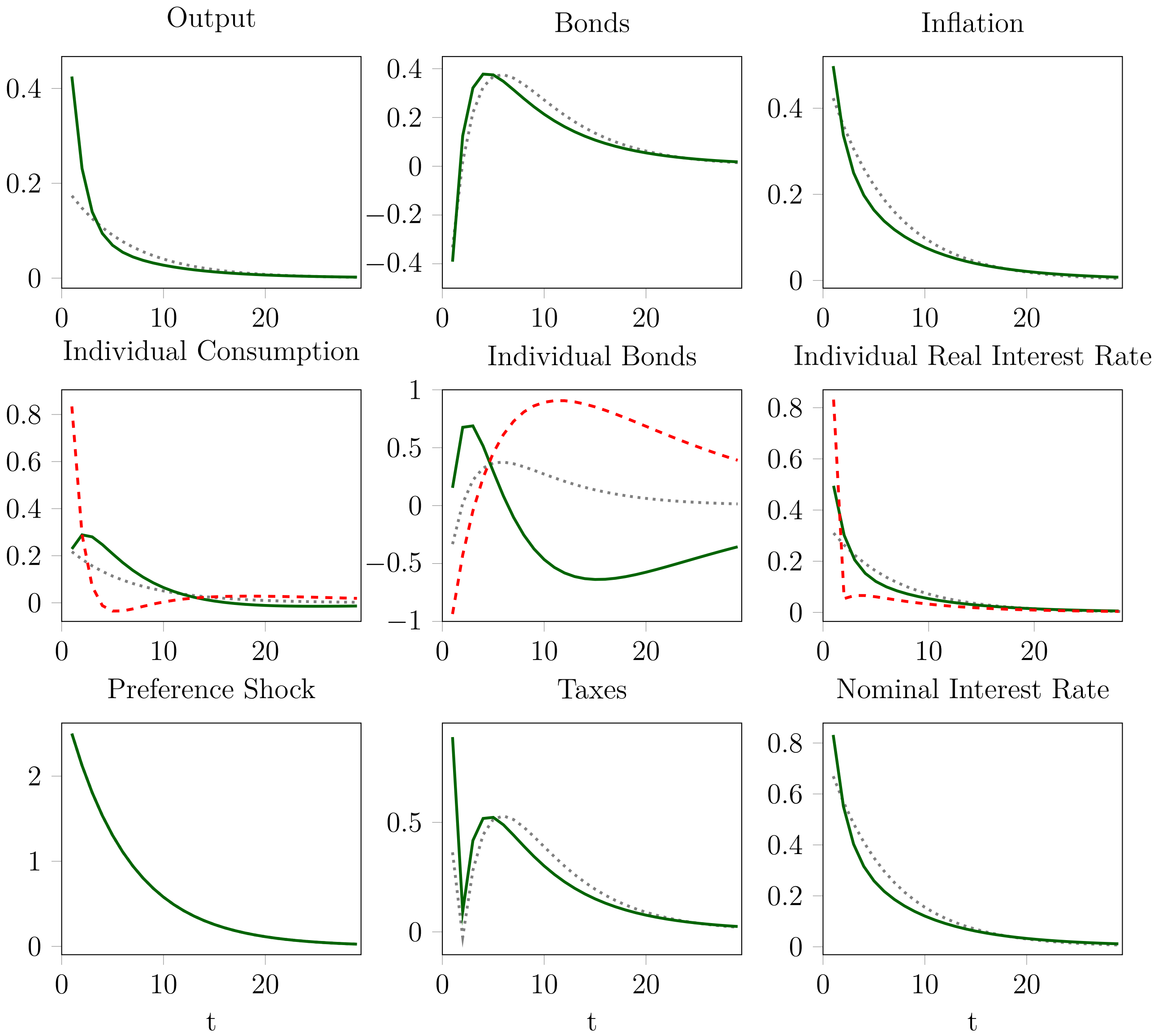

Figure 2: Impulse responses for a persistent 2.5% preference shock under baseline calibration

lower than would be the case if they did not anticipate a positive future preference shock (since $\upsilon_t - E\upsilon_{t+1} < \upsilon_t$).

On the other hand, the effects of the shock are quite different for boundedly rational agents. In the first period, their consumption decision will be subject to a preference shock, but the expectations that they form at the beginning of the period are not yet affected by the shock, so that $\upsilon_t - E^B\upsilon_{t+1} = \upsilon_t$ in that period. Thus, the initial consumption response will be far larger compared to rational agents as can be seen in the middle-left panel in Figure 2. However, from period 2 onward, boundedly rational agents will consistently overestimate the future shock realization and it will hold that $E^B\upsilon_{t+1} > E\upsilon_{t+1}$ for these periods. This preference shock bias induces boundedly rational agents to cut consumption more than rational agents from period 2 onward.

However, this effect is counteracted by the real interest rate bias and the consumption expectation bias. The subjective real interest rate of boundedly rational agents increases substantially more than the rational one in the first period as they expect next period's inflation to be zero. Also, output expectations are zero and thus below rational expectations. Hence, both biases reduce the effects of the shock on consumption of boundedly rational agents in the first period. The net effect of all three biases on boundedly rational consumption is however still substantially positive.

Further, form period 2 onward, the subjective real interest rate of boundedly rational agents is lower than that of rational agents since boundedly rational agents overestimate inflation consistently as it falls. Additionally, output expectations and therefore their own consumption expectations will be biased upwards as output falls. Both, the real interest rate bias and the consumption expectations bias therefore put upward pressure on consumption in these periods. However, also here the preference shock bias dominates so that boundedly rational consumption is lower than rational consumption.

Moreover, just as in the case of a cost-push shock, boundedly rational agents start to build up more real bond holdings in the medium-run (as can be seen in the middle panel of Figure 2), so that the wealth differences induced by the preference shock display more bond holdings for rational agents in the short-run, but more bond holdings for boundedly rational agents in the medium-run. Boundedly rational agents are then able to consume their savings later on so that their consumption eventually slightly overshoots the consumption of rational agents. However, note again that boundedly rational agents loose relatively more utility compared to rational agents over the life-cylce.

4 The rationality bias and monetary policy

So far, we have decomposed the rationality bias in three individual components, and have analyzed the effects of these biases on the micro and macro dynamics. In this section, we study the interaction between monetary policy and the rationality bias.

4.1 Eliminating the rationality bias

In turns out that the magnitude of the rationality bias directly depends on monetary policy. In particular, depending on monetary policy, the rationality bias can completely be eliminated. This is stated in Proposition 1.

Proposition 1. *If the log-deviations from steady state of the nominal interest rate and the inflation rate are equal in every period, then* $c^R_{t-1} = c^B_{t-1}$ *and* $b^B_{t-1} = b_{t-1}$ *imply that* $c^B_{t+s} = c^R_{t+s}$ *and* $b^B_{t+s-1} = b_{t+s-1}$*,* $s > 0$*. That is, shocks to the economy do not lead to a rationality bias for any parameterization of the model.*

Proof. When the nominal interest rate equals inflation in every period, we can write the Euler equation, (12), of rational agents as

$$c^R_t = E_t c^R_{t+1} - \frac{1}{\sigma}(\pi_t - E_t\pi_{t+1} - \upsilon_t + E_t\upsilon_{t+1}) = E_t c^R_{t+s} - \frac{1}{\sigma}(\pi_t - E_t\pi_{t+s} - \upsilon_t + E_t\upsilon_{t+s}), s > 0, \quad (32)$$

where the second equality follows from iterating this equation forward until period s.

Taking the limit $s \to \infty$, and using that $\lim_{s\to\infty} E_t c^R_{t+s} = \lim_{s\to\infty} E_t\pi_{t+s} = \lim_{s\to\infty} E_t\upsilon_{t+s} = 0$, it follows that

$$c^R_t = -\frac{1}{\sigma}\left(\pi_t - \upsilon_t\right). \quad (33)$$

For boundedly rational consumption, we use (18). When $\pi_t = i_t$, and assuming $c^R_{t-1} = c^B_{t-1}$ and $b^B_{t-1} = b_{t-1}$, we can write this as

$$c^B_t = \frac{1}{1-\bar{g}}(y_{t-1} - \hat{g}_{t-1}) - \frac{1}{\sigma}[\pi_t - \pi_{t-1} - \upsilon_t + \upsilon_{t-1}]. \tag{34}$$

Since it is assumed that $c^B_{t-1} = c^R_{t-1} = -\frac{1}{\sigma}(\pi_{t-1} - \upsilon_{t-1})$, it follows from market clearing that $y_{t-1} = -\frac{1}{\sigma}(1-\bar{g})(\pi_{t-1} - \upsilon_{t-1}) + \hat{g}_{t-1}$. Therefore, (34) reduces to

$$c^B_t = -\frac{1}{\sigma}(\pi_t - \upsilon_t). \tag{35}$$

This implies that when $c^R_{t-1} = c^B_{t-1}$ and $b^B_{t-1} = b_{t-1}$, it holds that $c^R_t = c^B_t$, so that boundedly rational and rational agents make identical decisions and also $b^B_t = b_t$. That is, no differences in consumption and wealth in period $t-1$ imply no consumption and wealth differences in period t. This then holds for all periods $s \geq t$, which proves the proposition. □

To understand the intuition of Proposition 1, recall the different components of the rationality bias depicted in Equation (31). As derived in the proof of the proposition, the rational consumption expectation is $E_t c^R_{t+1} = -\frac{1}{\sigma}E_t(\pi_{t+1} - \upsilon_{t+1})$. Also, using that initially there were no consumption and wealth heterogeneity (as in steady state) so that market clearing and Equation (33) imply $y_{t-1} = -\frac{1}{\sigma}(1-\bar{g})(\pi_{t-1} - \upsilon_{t-1}) + \hat{g}_{t-1}$, reduces Equation (31) to

$$\Delta_i c^i_t = c^B_t - c^R_t = -\frac{1}{\sigma}\Big(\pi_{t-1} - \upsilon_{t-1} - E_t(\pi_{t+1} - \upsilon_{t+1})\Big) \tag{36}$$

$$+\frac{1}{\sigma}(\pi_{t-1} - E_t\pi_{t+1}) - \frac{1}{\sigma}(\upsilon_{t-1} - E_t\upsilon_{t+1}) = 0. \tag{37}$$

In Equation (37) it can be seen that the consumption expectation bias, the real interest rate bias and the preference shock bias *exactly off-set* each other, even though they are different from zero *individually*. Hence, the above proposition implies that boundedly rational agents act *as if* they were rational, so that the difference $\Delta_i c^i_t$ is zero, but that they do so for different reasons than rational agents. Moreover, this holds for any fraction of rational agents, α.

Note that the assumption that the model is initialized with no consumption and wealth heterogeneity is not as restrictive as it may seem. In particular, since our model is stable and determinate, heterogeneity would, in the absence of shocks, always disappear over time as the model converges to steady state. Moreover, since Proposition 1 states that, when its conditions are satisfied, shocks do not lead to a rationality bias, it is intuitive that also in the presence of shocks consumption and wealth heterogeneity would disappear over time when $i_t = \pi_t$ in every period. We find that this is indeed the case. When we initialize simulations of the model with wealth and consumption heterogeneity and shock the model in every period, differences in consumption and bond holdings between the two agent types quickly go to zero in that case.

One possible implementation of Proposition 1 is setting $\phi_\pi = 1$ and $\phi_y = 0$ in the Taylor rule. However, that would imply that the model is (infinitely close to) indeterminate. Under some conditions, though, it is also possible to implement a monetary policy that results in an interest rate equal to the inflation rate with different combinations of ϕ_π and ϕ_y. This is stated in Proposition 2.

Proposition 2. *If the only shock in the economy affects inflation and output proportionally in every period, then for every value of ϕ_π a value of ϕ_y can be found that implements $i_t = \pi_t$ in every period. In particular:*

$$\phi_y = (1 - \phi_\pi)\frac{\pi_t}{y_t} \tag{38}$$

Proof. Plugging in $i_t = \pi_t$ in the Taylor rule, (26), and solving for ϕ_y, gives (38). This implies that when the ratio $\frac{\pi_t}{y_t}$ is constant over time, $i_t = \pi_t$ can be implemented in every period with constant coefficients ϕ_π and ϕ_y that satisfy this equation. □

Below we will see that the results of Proposition 2 can be used to implement Proposition 1 with $\phi_\pi > 1$ and $\phi_y > 0$ when the economy is hit by cost-push shocks, but that $\phi_y < 0$ would be required under preference shocks.

4.2 Implementation with cost-push shocks

First, consider the case where the only shocks in our model are cost-push shocks. We numerically search for combinations of ϕ_π and ϕ_y that minimize consumption and wealth heterogeneity and implement Propositions 1 and 2.[3] We find that a continuum of pairs $\{\phi_\pi, \phi_y\}$ eliminate the

[3] To that end, we calculate theoretical variances based on the MSV-solution of our model. Based on these theoretical variances, we then search, for different given values of ϕ_π, for the value of ϕ_y (in a grid) that minimizes

rationality bias. Moreover the ratio $\frac{\pi_t}{y_t}$ is constant and equal to -2.5 for all these pairs $\{\phi_\pi, \phi_y\}$, and they all satisfy (38). Moreover, the impulse responses of $\{\phi_\pi, \phi_y\}$ pairs that eliminate the rationality bias are all the same and are plotted in Figure 3.

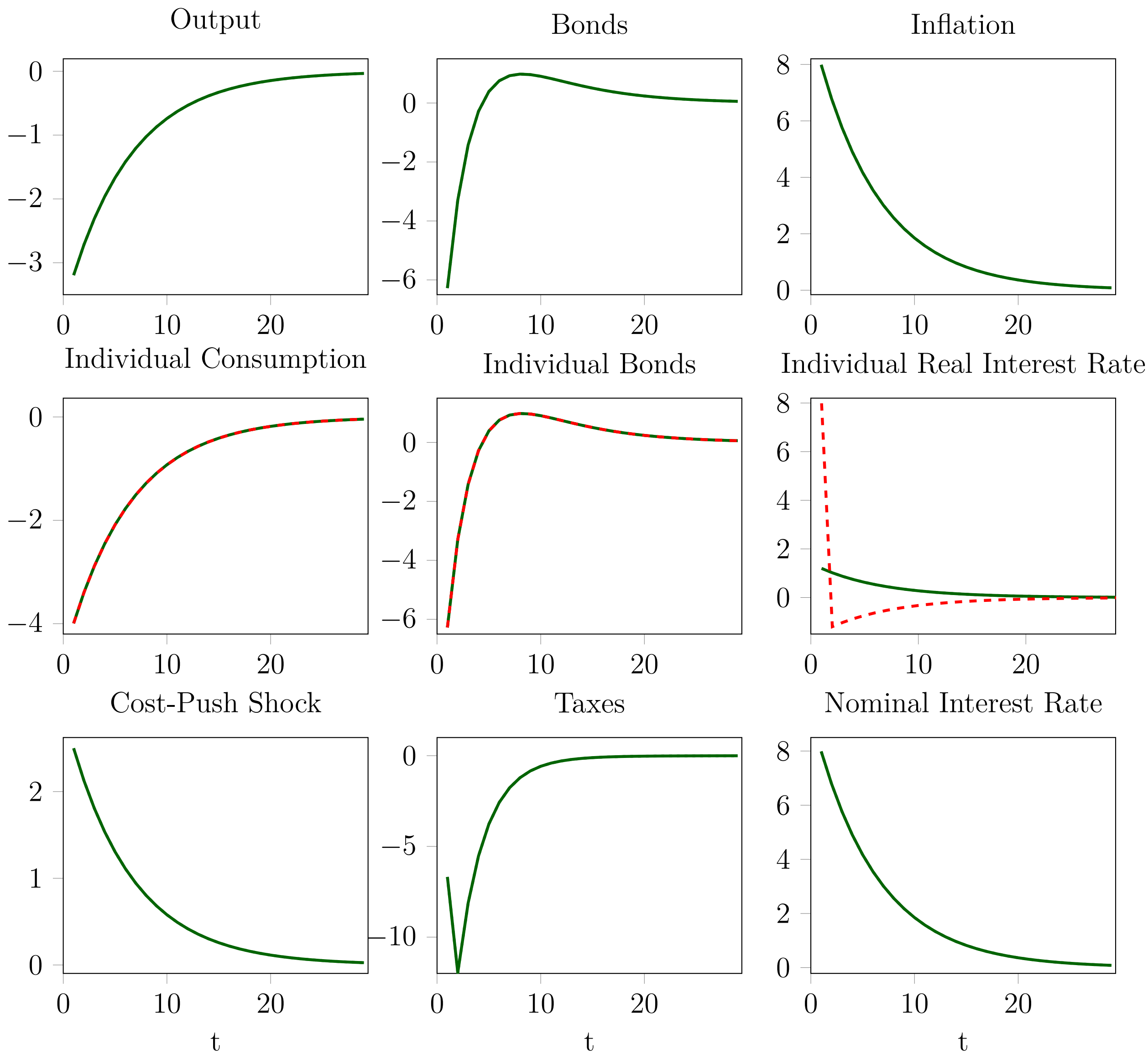

Figure 3: Representative impulse response where the rationality bias is eliminated under a cost-push shock

Comparing with the dynamics under the baseline parameterization in Figure 1, we can observe that although the rationality bias is eliminated, there still is a difference between the individual real interest rates, i.e. the real interest rate bias still appears, which is however exactly offset by the consumption expectation bias as discussed in Section 4.1.

Moreover, the elimination of the rationality bias implies a reduction in output volatility while

heterogeneity in wealth and consumption.

inflation becomes substantially more volatile. Under the baseline calibration we assumed $\phi_\pi = 1.5$ and $\phi_y = 0.2$. However, to satisfy (38), $\phi_\pi = 1.5$ would imply a very high output coefficient of $\phi_y = 1.25$ in the Taylor rule. Hence, the elimination of consumption and wealth heterogeneity requires a far stronger output stabilization than under the baseline calibration. As there is a trade-off between output and inflation variance in case of a cost-push shock, a strong reaction to output by the central bank implies higher inflation volatility. More realistic parameterizations that also implement the proposition would e.g. be $\phi_\pi = 1.1$ and $\phi_y = 0.25$ or $\phi_\pi = 1.2$ and $\phi_y = 0.5$. These parameterizations, however, imply the same inflation-output trade-off and relatively strong output stabilization.

4.3 Implementation with preference shocks

Next, we turn to preference shocks. Since inflation and output respond with the same sign to a preference shock, the ratio $\frac{\pi_t}{y_t}$ in (38) will be positive when the economy is hit by this shock. This implies that according to Proposition 2 a coefficient $\phi_\pi > 1$ requires a negative response to output to eliminate the rationality bias. This is not desirable and can lead to indeterminacy problems.

If, however, the central bank is assumed to be able to observe and respond to the shock, then an alternative way of implementing $i_t = \pi_t$ that does not lead to concerns for indeterminacy becomes available. This is stated in Proposition 3.

Proposition 3. *If inflation and output gap respond proportionally in every period to a shock (z_t), then $i_t = \pi_t$ can be achieved in every period if the central bank lets the interest rate correctly respond to the shock (with coefficient ϕ_z). In particular, for every combination of ϕ_π and ϕ_y, $i_t = \pi_t$ is achieved if*

$$i_t = \phi_\pi \pi_t + \phi_y y_t + \phi_z z_t, \tag{39}$$

with

$$\phi_z = (1 - \phi_\pi)\frac{\pi_t}{z_t} - \phi_y \frac{y_t}{z_t} \tag{40}$$

Proof. Plugging in $i_t = \pi_t$ in (39) and solving for ϕ_z gives (40). This implies that when the ratios $\frac{\pi_t}{z_t}$ and $\frac{y_t}{z_t}$ are constant over time, $i_t = \pi_t$ can be implemented in every period with constant coefficients ϕ_π and ϕ_y that satisfy this equation. □

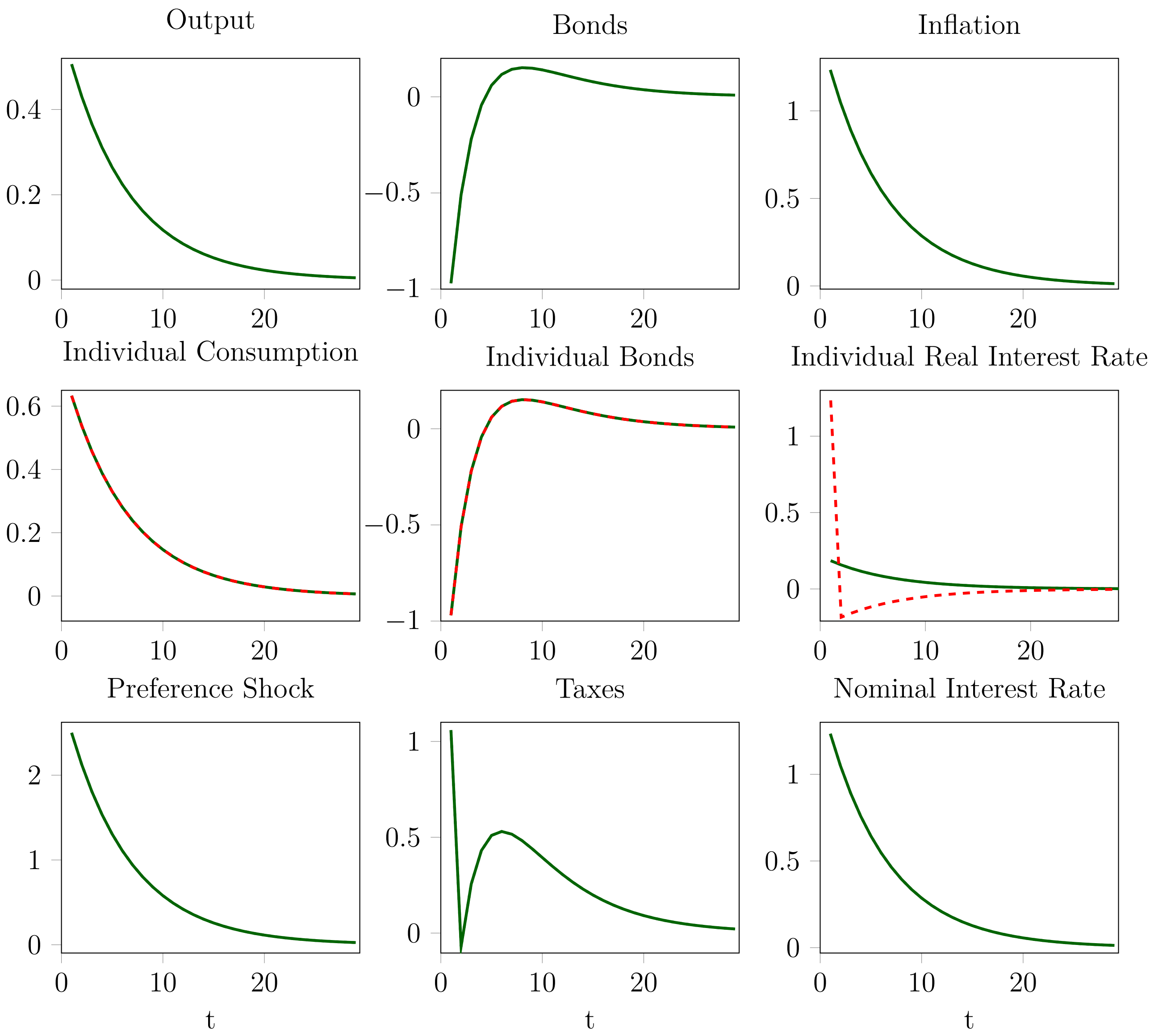

Figure 4: Representative impulse response where the rationality bias is eliminated under a preference shock

Since under homogeneous rational expectations ($\alpha = 1$), our model contains no endogenous state variables, the ratios $\frac{\pi_t}{z_t}$ and $\frac{y_t}{z_t}$ will be constant in that case. Moreover, when the rationality bias is eliminated, boundedly rational agents act as if they were rational, so that the model behaves as under homogeneous rational expectations. Hence, when ϕ_z is chosen according to (40), the conditions of Proposition 3 are satisfied, and the rationality bias is eliminated. This holds for any combination of ϕ_π and ϕ_y and for any value of α.

Similar to the case of the cost-push shock, we numerically search for combinations of ϕ_π, ϕ_y and ϕ_z that minimize consumption and wealth heterogeneity and implement Propositions 1 and 3. We again find that the rationality bias can be eliminated with a continuum of policy parameter combinations. Moreover the ratio $\frac{\pi_t}{z_t}$ is found to be constant and equal to 0.49 for all these parameter combinations, while $\frac{y_t}{z_t}$ is equal to 0.2. Under the baseline calibration with $\phi_\pi = 1.5$ and $\phi_y = 0.2$ the rationality bias is therefore eliminated with $\phi_z = -0.29$. The impulse responses are again the same for all parameter combinations that implement Proposition 3 and are plotted in Figure 4.

Comparing Figure 4 with the impulse responses under the baseline calibration in Figure 2 shows that eliminating the rationality bias comes at the cost of slightly higher volatility in output but also significantly higher volatility in inflation. Thus, in case of a preference shock, minimizing the rationality bias comes at the expense of higher aggregate volatility, especially in inflation.

4.4 Welfare

We found that the rationality disappears under certain monetary policy specifications, but that such policy implies more inflation volatility both under the cost-push shock and under the preference shock. It, therefore, may not be desirable for a central bank to implement a policy where the bias is completely eliminated.

In Appendix A.6 we show that, under the assumption that individual preferences can be aggregated by simply summing up their utilities, the second-order approximated aggregate utility loss is given by

$$L_t \simeq a_1 var(y_t) + a_2 var(\pi_t) + a_3 var(c_t^B - c_t^R) \tag{41}$$

with

$$a_1 = \left[\gamma + \frac{\sigma}{(1-\bar{g})}\right] \tag{42}$$

$$a_2 = \frac{\eta}{\delta} \tag{43}$$

$$a_3 = \alpha(1-\alpha)\left[(1-\bar{g})\sigma + \frac{\sigma^2}{\gamma^2}\right]. \tag{44}$$

Under the baseline calibration, the weight a_3 on the variance of the rationality bias is approximately three percent of the weight on inflation volatility.[4] This reflects a very strong distaste of

[4]the weight on output volatility, a_1, is approximately five percent of that on inflation volatility under the benchmark calibration.

agents in this economy for inflation volatility and a rather minor distaste for consumption heterogeneity (and output volatility). Thus, even though the third term in Equation (41) disappears when Proposition 1 is implemented, the corresponding increase in inflation volatility will lead to a considerably higher welfare loss. Hence, based on (41), agents in this economy will accept a certain rationality bias and thus a certain heterogeneity in consumption (and wealth) if this comes with a policy that achieves price stability. Note that the strong dislike for inflation volatility is model-inherent and robust with respect to the parameterization.[5]

Of course, if a social planer believes that reducing the rationality bias and therefore consumption and wealth heterogeneity is more important than is reflected in the utility functions of the agents populating the economy, a different conclusion may be reached. Our findings regarding welfare are very much in line with the results in Debortoli and Galí (2017), who consider a two-agent New Keynesian (TANK) model where the weight in the central bank's micro-founded loss function on a certain heterogeneity index is equivalently low.

5 Conclusion

We build a macroeconomic model with different degrees of rationality. While rational agents are indeed fully rational, boundedly rational agents are assumed to be considerably less sophisticated. Boundedly rational agents follow the less cognitive demanding Euler-equation learning and use a simple naive forecast heuristic to form expectations.

Because both agent types make different decisions, substantial consumption and wealth heterogeneity arises when the economy is hit by shocks. We show that the rationality bias of boundedly rational agents is the driver of consumption and wealth heterogeneity and can be decomposed into three different components: the consumption expectation bias, the real interest rate bias and the preference shock expectation bias. Further, we show that certain monetary policy can eliminate this bias independent of the shock type. In this case the components of the rationality bias exactly offset each other while they are non-zero individually.

However, it might not be desirable to from a welfare perspective to eliminate the rationality

[5] See e.g. Galí (2008), Woodford (2003), Di Bartolomeo et al. (2016) and Debortoli and Galí (2017) for loss functions in similar models.

bias and therefore consumption and wealth heterogeneity as this comes along with high inflation volatility.

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A Microfoundations

A.1 Consumption of rational agents

Recall the log-linearized budget constraint of rational agents

$$\hat{b}_t^R = h_t^R + w_t + \beta^{-1}\hat{b}_{t-1}^R + \bar{b}\beta^{-1}(i_{t-1} - \pi_t) + \eta^{-1}d_t - (1-\bar{g})c_t^R - \frac{\bar{T}}{\bar{Y}}\tau_t. \tag{A.1}$$

Iterating until infinity and rearranging gives

$$\begin{aligned} E_t^R \sum_{s=t}^{\infty} \beta^{s-t} c_s^R &= (\beta(1-\bar{g}))^{-1}\hat{b}_{t-1}^R \\ &+ E_t^R \sum_{s=t}^{\infty} \beta^{s-t}(1-\bar{g})^{-1}\Big[h_s^R + w_s + \bar{b}\beta^{-1}(i_{s-1} - \pi_s) + \eta^{-1}d_s - \frac{\bar{T}}{\bar{Y}}\tau_s\Big]. \end{aligned} \tag{A.2}$$

Using $h_t^R = \frac{1}{\gamma}(w_t - \sigma c_t^R)$ and solving for c_s^R yields

$$\begin{aligned} \Big[\frac{\gamma + \sigma(1-\bar{g})^{-1}}{\gamma}\Big] E_t^R \sum_{s=t}^{\infty} \beta^{s-t} c_s^R &= (\beta(1-\bar{g}))^{-1}\hat{b}_{t-1}^R \\ + \sum_{s=t}^{\infty} \beta^{s-t}(1-\bar{g})^{-1}\Big[\frac{\gamma+1}{\gamma}w_s + \bar{b}\beta^{-1}(i_{s-1} - \pi_s) + \eta^{-1}d_s - \frac{\bar{T}}{\bar{Y}}\tau_s\Big]&. \end{aligned} \tag{A.3}$$

For now we are focusing on the left-hand side of (A.3) which can be written as

$$\Big[\frac{\gamma + \sigma(1-\bar{g})^{-1}}{\gamma}\Big] c_t^R + \Big[\frac{\gamma + \sigma(1-\bar{g})^{-1}}{\gamma}\Big] E_t^R \sum_{s=t+1}^{\infty} \beta^{s-t} c_s^R. \tag{A.4}$$

Then we can iterate the Euler equation until period s

$$E_t^R c_s^R = c_t^R + \frac{1}{\sigma} E_t^R \sum_{k=t}^{s-1} (i_k - \pi_{k+1} - \upsilon_k + \upsilon_{k+1}). \tag{A.5}$$

Using this, we can write

$$\Big[\frac{\gamma+\sigma(1-\bar{g})^{-1}}{\gamma}\Big]E_t^R\sum_{s=t}^{\infty}\beta^{s-t}c_s^R=\Big[\frac{\gamma+\sigma(1-\bar{g})^{-1}}{(1-\beta)\gamma}\Big]c_t^R$$
$$+\Big[\frac{\gamma+\sigma(1-\bar{g})^{-1}}{\gamma\sigma}\Big]E_t^R\sum_{s=t+1}^{\infty}\sum_{k=t}^{s-1}\beta^{s-t}(i_k-\pi_{k+1}-\upsilon_k+\upsilon_{k+1}). \tag{A.6}$$

Simplifying the double sum yields

$$\Big[\frac{\gamma+\sigma(1-\bar{g})^{-1}}{\gamma}\Big]E_t^R\sum_{s=t}^{\infty}\beta^{s-t}c_s^R=\Big[\frac{\gamma+\sigma(1-\bar{g})^{-1}}{(1-\beta)\gamma}\Big]c_t^R$$
$$+\Big[\frac{\gamma+\sigma(1-\bar{g})^{-1}}{\gamma\sigma}\Big]\frac{\beta}{1-\beta}E_t^R\sum_{s=t}^{\infty}\beta^{s-t}(i_s-\pi_{s+1}-\upsilon_s+\upsilon_{s+1}). \tag{A.7}$$

Inserting into (A.3) and solving for c_t^R gives

$$\begin{aligned}c_t^R=&\zeta\hat{b}_{t-1}^R+\zeta\bar{b}(i_{t-1}-\pi_t) \qquad \text{(A.8)}\\&+\zeta\beta E_t^R\sum_{s=t}^{\infty}\beta^{s-t}[\frac{\gamma+1}{\gamma}w_s+\eta^{-1}d_s-\frac{\bar{T}}{\bar{Y}}\tau_s]\\&-\frac{(1-\zeta\bar{b}\sigma)\beta}{\sigma}E_t^R\sum_{s=t}^{\infty}\beta^{s-t}(i_s-\pi_{s+1})+\frac{\beta}{\sigma}E_t^R\sum_{s=t}^{\infty}\beta^{s-t}(\upsilon_s-\upsilon_{s+1})\end{aligned}$$

with $\zeta=\frac{(1-\beta)\gamma}{(\gamma(1-\bar{g})+\sigma)\beta}$. Taking $w_t=\gamma h_t^i+\sigma c_t^i$, integrating over agent types and using $y_t=h_t$ and $c_t=\frac{y_t}{1-\bar{g}}-\frac{\hat{g}_t}{1-\bar{g}}$ gives $w_t=(\gamma+\frac{\sigma}{1-\bar{g}})y_t-\sigma\frac{\hat{g}_t}{1-\bar{g}}$. Further, total dividends are defined as $D_t=Y_t-(1-s)W_tH_t$. Log-Linearizing and using $y_t=h_t$ gives

$$d_t=\frac{\bar{Y}}{\bar{D}}[(1-(1-s)\bar{W})y_t-(1-s)\bar{W}w_t]. \tag{A.9}$$

Using $w_t=(\gamma+\frac{\sigma}{1-\bar{g}})y_t-\sigma\frac{\hat{g}_t}{1-\bar{g}}$, $(1-s)\bar{W}=1-\eta^{-1}$ and $\bar{D}=\bar{Y}\eta^{-1}$ yields

$$d_t=[1-(\eta-1)(\gamma+\frac{\sigma}{1-\bar{g}})]y_t+(\eta-1)\sigma\frac{\hat{g}_t}{1-\bar{g}}. \tag{A.10}$$

Using the expression for d_t and w_t and inserting into (A.8) gives

$$\begin{aligned} c_t^R &= \zeta \hat{b}_{t-1}^R + \zeta \bar{b}(i_{t-1} - \pi_t) + \zeta \beta E_t^R \sum_{s=t}^{\infty} \beta^{s-t}[\Gamma_y y_s - \Gamma_g \hat{g}_s - \Gamma_\tau \tau_s] \\ &- \frac{(1-\zeta\bar{b}\sigma)\beta}{\sigma} E_t^R \sum_{s=t}^{\infty} \beta^{s-t}(i_s - \pi_{s+1}) + \frac{\beta}{\sigma} E_t^R \sum_{s=t}^{\infty} \beta^{s-t}(\upsilon_s - \upsilon_{s+1}) \end{aligned} \quad \text{(A.11)}$$

with

$$\Gamma_y = \frac{1+\gamma}{\eta} + 1 + \frac{(\gamma+\eta)\sigma}{\gamma\eta(1-\bar{g})} \quad \text{(A.12)}$$

$$\Gamma_g = \frac{(\gamma+\eta)\sigma}{(1-\bar{g})\gamma\eta} \quad \text{(A.13)}$$

$$\Gamma_\tau = \frac{\bar{T}}{\bar{Y}}. \quad \text{(A.14)}$$

Further, Equation (A.1) can be rewritten as

$$\hat{b}_t^R = \beta^{-1}\hat{b}_{t-1}^R - \Gamma_c c_t^R + \Gamma_y y_t + \bar{b}\beta^{-1}(i_{t-1} - \pi_t) - \Gamma_g \hat{g}_t - \Gamma_\tau \hat{\tau}_t \quad \text{(A.15)}$$

with $\Gamma_c = (1-\bar{g}) + \frac{\sigma}{\gamma}$. Equation (A.15) can equivalently be obtained for boundedly rational agents.

A.2 Consumption of boundedly rational agents

The consumption Euler equation for boundedly rational agents reads

$$c_t^B = E_t^B c_{t+1}^B - \frac{1}{\sigma}(i_t - E_t^B \pi_{t+1} - \upsilon_t + \upsilon_{t+1}). \quad \text{(A.16)}$$

Iterated forward until period N which is far enough in the future for the boundedly rational agents to consider it as "long-run" (but not infinity) and using the law of iterated expectations (LIE) gives

$$c_t^B = E_t^B c_N^B - \frac{1}{\sigma} \sum_{k=0}^{N} E_t^B [i_{t+k} - \pi_{t+k+1} - \upsilon_{t+k} + \upsilon_{t+k+1}]. \quad \text{(A.17)}$$

Also, agents are assumed to know about market clearing $y_t = (1-\bar{g})c_t + \hat{g}_t$ which can be written one period ahead from the point of view of boundedly rational agents as

$$E_t^B y_{t+1} = (1-\bar{g})E_t^B(\alpha c_{t+1}^R + (1-\alpha)c_{t+1}^B) + E_t^B \hat{g}_{t+1}. \tag{A.18}$$

Further, we assume that boundedly rational agents also be believe rational agents to satisfy their consumption Euler equation and that they iterate it until period N. Inserting both forward-iterated individual consumption Euler equations yields

$$\begin{aligned} E_t^B y_{t+1} = (1-\bar{g})E_t^B \Big(& \alpha(E_{t+1}^R c_N^R - \frac{1}{\sigma}\sum_{k=1}^{N} E_{t+1}^R[i_{t+k} - \pi_{t+k+1} - \upsilon_{t+k} + \upsilon_{t+k+1}]) + \\ & (1-\alpha)(E_{t+1}^B c_N^B - \frac{1}{\sigma}\sum_{k=1}^{N} E_{t+1}^B[i_{t+k} - \pi_{t+k+1} - \upsilon_{t+k} + \upsilon_{t+k+1}]\Big) + E_t^B \hat{g}_{t+1}. \end{aligned} \tag{A.19}$$

This can be written using Branch and McGough (2009)'s assumptions on higher-order beliefs as

$$E_t^B \frac{y_{t+1}}{1-\bar{g}} - E_t^B c_N - E_t^B \frac{\hat{g}_{t+1}}{1-\bar{g}} = -\frac{1}{\sigma} E_t^B \sum_{k=1}^{N} [i_{t+k} - \pi_{t+k+1} - \upsilon_{t+k} + \upsilon_{t+k+1}]. \tag{A.20}$$

Further, writing (A.17) as

$$c_t^B - E_t^B c_N^B + \frac{1}{\sigma}[i_t - E_t^B \pi_{t+1} - \upsilon_t + \upsilon_{t+1}] = -\frac{1}{\sigma} E_t^B \sum_{k=1}^{N} [i_{t+k} - \pi_{t+k+1} - \upsilon_{t+k} + \upsilon_{t+k+1}] \tag{A.21}$$

and equating (A.20) and (A.21) gives

$$c_t^B = \frac{1}{1-\bar{g}} E_t^B y_{t+1} + E_t^B(c_N^B - c_N) - \frac{1}{\sigma}[i_t - E_t^B \pi_{t+1} - \upsilon_t + E_t^B \upsilon_{t+1}] - \frac{1}{1-\bar{g}} E_t^B \hat{g}_{t+1}. \tag{A.22}$$

Assuming that, when boundedly rational agents have more wealth than the average, they will be able to consume more than the average in the long-run, i.e. $E_t^B(c_N^B - c_N) = \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1})$, yields

$$c_t^B = \frac{1}{1-\bar{g}} E_t^B y_{t+1} + \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1}) - \frac{1}{\sigma}[i_t - E_t^B \pi_{t+1} - \upsilon_t + \upsilon_{t+1}] - \frac{1}{1-\bar{g}} E_t^B \hat{g}_{t+1}. \tag{A.23}$$

A.3 Aggregation of individual consumption decisions

Plugging individual consumption decisions (17) and (A.11) into market clearing $y_t = (1-\bar{g})(\alpha c_t^R + (1-\alpha)c_t^B) + \hat{g}_t$ and using $E_t^R = E_t$ and $E_t^B x_{t+1} = x_{t-1}$ gives

$$\begin{aligned} y_t = (1-\bar{g})\big[&\alpha\zeta\hat{b}_{t-1}^R + \alpha\zeta\bar{b}(i_{t-1}-\pi_t) + \zeta\alpha\beta E_t \sum_{s=t}^{\infty}\beta^{s-t}[\Gamma_y y_s - \Gamma_g \hat{g}_s - \Gamma_\tau \tau_s] \\ &- \frac{(1-\zeta\bar{b}\sigma)\alpha\beta}{\sigma} E_t \sum_{s=t}^{\infty}\beta^{s-t}(i_s - \pi_{s+1}) + \frac{\alpha\beta}{\sigma} E_t^R \sum_{s=t}^{\infty}\beta^{s-t}(\upsilon_s - \upsilon_{s+1}) \\ &+ (1-\alpha)(1-\bar{g})^{-1} y_{t-1} + (1-\alpha)\psi(\hat{b}_{t-1}^B - \hat{b}_{t-1}) \\ &- \frac{(1-\alpha)}{\sigma}[i_t - \pi_{t-1} - \upsilon_t + \upsilon_{t-1}] - (1-\alpha)(1-\bar{g})^{-1}\hat{g}_{t-1}\big] + \hat{g}_t. \end{aligned} \tag{A.24}$$

Writing (A.24) one period ahead (from the point of view of rational agents) and multiplying by β and substracting the resulting equation from (A.24) gives after collecting terms and solving for y_t

$$\begin{aligned} y_t = \;& \Phi_1 E_t y_{t+1} + \Phi_2 y_{t-1} + \Phi_3 E_t \pi_{t+1} - \Phi_4 \pi_t + \Phi_5 \pi_{t-1} + \Phi_6 i_{t-1} - \Phi_7 i_t \\ &+ \Phi_8 E_t i_{t+1} - \Phi_9 \hat{b}_{t-1}^R + \Phi_{10} b_t^R - \Phi_{11}\hat{g}_{t-1} + \Phi_{12}\hat{g}_t - \Phi_{13} E_t \hat{g}_{t+1} \\ &- \Phi_{14}\tau_t + \Phi_{15}\hat{b}_{t-1} - \Phi_{16}\hat{b}_t + \Phi_{17}\upsilon_t - \Phi_{18} E_t \upsilon_{t+1} - \Phi_5 \upsilon_{t-1} \end{aligned} \tag{A.25}$$

with

$$\Phi_1 = \frac{\beta}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.26}$$

$$\Phi_2 = \frac{(1-\alpha)}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.27}$$

$$\Phi_3 = \frac{(1-\bar{g})\alpha\beta}{(1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta)\sigma} \tag{A.28}$$

$$\Phi_4 = \frac{(1-\bar{g})((1-\alpha)\beta + \bar{b}\alpha\zeta\sigma)}{(1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta)\sigma} \tag{A.29}$$

$$\Phi_5 = \frac{(1-\bar{g})(1-\alpha)}{(1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta)\sigma} \tag{A.30}$$

$$\Phi_6 = \frac{(1-\bar{g})\alpha\zeta\bar{b}}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.31}$$

$$\Phi_7 = \frac{(1-\bar{g})(1-\alpha(1-\beta))}{(1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta)\sigma} \tag{A.32}$$

$$\Phi_8 = \frac{(1-\bar{g})(1-\alpha)\beta}{(1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta)\sigma} \tag{A.33}$$

$$\Phi_9 = \frac{(1-\bar{g})\alpha(\psi-\zeta)}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.34}$$

$$\Phi_{10} = \frac{(1-\bar{g})\alpha\beta(\psi-\zeta)}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.35}$$

$$\Phi_{11} = \frac{(1-\alpha)}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.36}$$

$$\Phi_{12} = \frac{1-(1-\bar{g})\alpha\beta\Gamma_g\zeta + (1-\alpha)\beta}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.37}$$

$$\Phi_{13} = \frac{\beta}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.38}$$

$$\Phi_{14} = \frac{(1-\bar{g})\alpha\beta\zeta\Gamma_\tau}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.39}$$

$$\Phi_{15} = \frac{(1-\bar{g})\alpha\psi}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.40}$$

$$\Phi_{16} = \frac{(1-\bar{g})\alpha\psi\beta}{1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta} \tag{A.41}$$

$$\Phi_{17} = \frac{(1-\bar{g})((1-\alpha)(1+\beta)+\alpha\beta)}{(1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta)\sigma} \tag{A.42}$$

$$\Phi_{18} = \frac{(1-\bar{g})\beta}{(1-(1-\bar{g})\alpha\beta\Gamma_y\zeta + (1-\alpha)\beta)\sigma}. \tag{A.43}$$

A.4 Phillips-Curve

Log-linearizing firms optimality condition (8) around the zero-inflation steady state gives

$$\tilde{p}_t = (1-\omega\beta)mc_t + \omega\beta E_t[\tilde{p}_{t+1} + \pi_{t+1}] \tag{A.44}$$

where marginal costs are given by

$$mc_t = w_t = (\gamma + \frac{\sigma}{1-\bar{g}})y_t - \sigma\frac{\hat{g}_t}{1-\bar{g}} + \mu_t. \tag{A.45}$$

where μ_t is a time varying exogenous wage markup (as in Galí, 2008), which we refer to as a cost-push shock. Plugging in marginal costs in (A.44) and combining with the log-linearized evolution of aggregate prices $\pi_t = \frac{1-\omega}{\omega}\tilde{p}_t$ yields the Phillips-Curve given by (21).

A.5 Steady state

In this section, we derive the steady state around which the model is log-linearized, where gross inflation equals 1.

Evaluating (8) at the zero inflation steady state gives

$$\bar{MC} = \frac{\eta - 1}{\eta}. \tag{A.46}$$

From the first order conditions of the households it follows that in this steady state we must have

$$1 + \bar{i} = \frac{1}{\beta}. \tag{A.47}$$

Furthermore, it follows from Equation 5 that

$$\bar{H} = \bar{Y}. \tag{A.48}$$

Next, we solve the steady state aggregate resource constraint for consumption, and write

$$\bar{C} = \bar{Y}(1 - \bar{g}). \tag{A.49}$$

Plugging in these steady state labor and consumption levels in (4) and using $s = \frac{1}{\eta}$ gives

$$\bar{W} = \bar{Y}^{\gamma}(\bar{Y}(1 - \bar{g}))^{\sigma} = \bar{Y}^{\gamma+\sigma}(1 - \bar{g})^{\sigma} = \frac{1}{(1 - s)}\bar{MC} = 1. \tag{A.50}$$

Where the last equality follows from $\bar{MC} = (1 - s)\bar{W}$ and (A.46). We can thus write

$$\bar{Y} = \left(\frac{1}{1 - \bar{g}}\right)^{\frac{\sigma}{\gamma+\sigma}}. \tag{A.51}$$

Then we turn to the government budget constraint. In steady state, (9) reduces to

$$\bar{b} = \frac{\bar{g} - \frac{\bar{T}}{\bar{Y}} + \eta^{-1}}{1 - \beta^{-1}} \tag{A.52}$$

where we use $s = \frac{1}{\eta}$ and substitute for the interest rate using (A.47).

A.6 Derivation of second-order approximated utilitarian welfare loss

Household utility is given by

$$\Upsilon_t U_t^i = U(C_t^i, \Upsilon_t) - V(H_t^i, \Upsilon_t) = \frac{\Upsilon_t (C_t^i)^{1-\sigma}}{1-\sigma} - \frac{\Upsilon_t (H_t^i)^{1+\gamma}}{1+\gamma} \tag{A.53}$$

Therefore, $U_C = (\bar{C})^{-\sigma}$, $U_{CC} = -\sigma U_C$, $U_{C\Upsilon} = U_C$ and $V_H = (\bar{Y})^{\eta}$, $V_{HH} = \eta V_H$, $V_{H\Upsilon} = V_H$. Further, we will use $U_C = \bar{W} V_H$ (labor-leisure condition) where $\bar{W} = 1$ due to a subsidy to marginal costs to firms so that the steady state is efficient. Thus, $U_C = V_H$. As before, lower case letter indicated a log-deviation from steady state. We will frequently use the fact that

$$\frac{X_t - \bar{X}}{\bar{X}} = x_t + \frac{1}{2} x_t^2 \tag{A.54}$$

holds up to second-order for any variable X. Hence, the second-order approximated goods market clearing condition in terms of log-deviations from steady state becomes $y_t + \frac{1}{2} y_t^2 = (1-\bar{g})(c_t + \frac{1}{2} c_t^2) + \mathcal{O}(3)$ where we already assume that government spending will not deviate from its steady state value. Taking squares, the latter can be written as $y_t^2 = (1-\bar{g})^2 c_t^2 + \mathcal{O}(3)$. Plugging back in yields $y_t = (1-\bar{g}) c_t + \frac{1}{2} \bar{g}(1-\bar{g}) c_t^2 + \mathcal{O}(3)$ which can be rewritten as

$$(1-\bar{g}) c_t = y_t - \frac{1}{2} \frac{\bar{g}}{(1-\bar{g})} y_t^2 + \mathcal{O}(3). \tag{A.55}$$

Now, we turn to approximating the utility function (A.53) using the results above. The consumption utility part in (A.53) can be approximated as (see Woodford, 2003 or Di Bartolomeo et al., 2016)

$$U(\cdot) = \bar{C} U_C \Big(c_t - \frac{\sigma}{2} var_i(c_t^i) + \frac{1-\sigma}{2} (c_t)^2 + \hat{\upsilon}_t c_t \Big) + t.i.p. + \mathcal{O}(3) \tag{A.56}$$

where we used that $c_t = E_i c_t^i + \frac{1}{2} var_i(c_t^i) + \mathcal{O}(3)$ holds up to a second order and where $t.i.p.$ contains steady state values and shock terms that are not interacting with policy dependent variables.

Using market clearing, Equation (A.56) can be written as

$$U(\cdot) = \bar{Y} U_C \Big(y_t - \frac{1}{2} \frac{\bar{g}}{(1-\bar{g})} y_t^2 - (1-\bar{g}) \frac{\sigma}{2} var_i(c_t^i) + \frac{1-\sigma}{2(1-\bar{g})} (y_t)^2 + \upsilon_t y_t \Big) + t.i.p. + \mathcal{O}(3). \tag{A.57}$$

which can be rewritten as

$$U(\cdot) = \bar{Y}U_C\Big(y_t - (1-\bar{g})\frac{\sigma}{2}var_i(c_t^i) + \frac{1-\frac{\sigma}{(1-\bar{g})}}{2}y_t^2 + \upsilon_t y_t\Big) + t.i.p. + \mathcal{O}(3). \tag{A.58}$$

Now, we turn to the approximation of the labor-utility term which is analogously given by

$$V(\cdot) = \bar{Y}V_H\Big(h_t + \frac{\gamma}{2}var_i(h_t^i) + \frac{1+\gamma}{2}h_t^2 + \upsilon_t h_t\Big) + t.i.p. + \mathcal{O}(3). \tag{A.59}$$

Now, our goal is to write aggregate labor in terms of aggregate output and price dispersion. To that end, we follow Galí (2008) where we use $H_t = \int_0^1 H_t(j) = \int_0^1 Y_t(j)$ which is equal to

$$H_t = Y_t \int_0^1 \Big(\frac{P_t(j)}{P_t}\Big)^{-\eta} dj. \tag{A.60}$$

Making a second-order approximation and using that $\frac{\eta}{2}var_i(p_t(i)) = log\Big(\int_0^1 \Big(\frac{P_t(j)}{P_t}\Big)^{-\eta} dj\Big)$ (Lemma 1 in Appendix A in Chapter 4, Galí, 2008) yields

$$h_t = y_t + \frac{\eta}{2}var_j(p_t(j)). \tag{A.61}$$

Inserting gives

$$v(\cdot) = \bar{Y}V_H\Big(y_t + \frac{\eta}{2}var_j(p_t(j)) + \frac{\gamma}{2}var_i(h_t^i) + \frac{1+\gamma}{2}(y_t)^2 + \upsilon_t y_t\Big) + t.i.p. + \mathcal{O}(3). \tag{A.62}$$

Now, using the linear labor-leisure condition (which we can do as we insert it into a variance which is already second-order) we can write $\frac{\gamma}{2}var_i(h_t^i) = \frac{\sigma^2}{2\gamma^2}var_i(c_t^i)$.[6] Hence, (A.62) becomes (also using $V_h = U_c$ from above)

$$V(\cdot) = \bar{Y}U_H\Big(y_t + \frac{\eta}{2}var_j(p_t(j)) + \frac{\sigma^2}{2\gamma^2}var_i(c_t^i) + \frac{1+\gamma}{2}(y_t)^2 + \upsilon_t y_t\Big) + t.i.p. + \mathcal{O}(3). \tag{A.63}$$

[6] $var_i(\frac{1}{\gamma}w_t - \frac{\sigma}{\gamma}c_t^i) = var_i(\frac{1}{\gamma}w_t) + var_i(\frac{\sigma}{\gamma}c_t^i) - 2Cov_i(\frac{1}{\gamma}w_t, \sigma c_t^i) = \frac{\sigma^2}{\gamma^2}var_i(c_t^i)$.

Inserting A.58 and A.63 into (A.53) gives

$$\bar{Y}U_C\Big(y_t - (1-\bar{g})\frac{\sigma}{2}var_i(c_t^i) + \frac{1-\frac{\sigma}{(1-\bar{g})}}{2}(y_t)^2 + \upsilon_t y_t\Big) \tag{A.64}$$

$$-\Big[\bar{Y}U_C\Big(y_t + \frac{\eta}{2}var_j(p_t(j)) + \frac{\sigma^2}{2\gamma^2}var_i(c_t^i) + \frac{1+\gamma}{2}(y_t)^2 + \upsilon_t y_t\Big)\Big] + t.i.p. + \mathcal{O}(3) \tag{A.65}$$

and combining

$$-\frac{\bar{Y}U_C}{2}\Big[a_1(y_t)^2 + \tilde{a}_2 var_j(p_t(j)) + \tilde{a}_3 var_i(c_t^i)\Big] + t.i.p. + \mathcal{O}(3) \tag{A.66}$$

where $a_1 = [\gamma + \frac{\sigma}{(1-\bar{g})}]$, $\tilde{a}_2 = \eta$ and $\tilde{a}_3 = [(1-\bar{g})\sigma + \frac{\sigma^2}{\gamma^2}]$. Further, $var_j(p_t(j)) = \delta^{-1}\pi_t^2$ with $\delta = \frac{(1-\omega)(1-\omega\beta)}{\omega}$ (see Woodford, 2003, Chapter 6) and $var_i(c_t^i) = \alpha(1-\alpha)(c_t^B - c_t^R)^2$. Thus, the aggregated intertemporal utility can be can written as

$$\mathcal{W} = -\frac{\bar{Y}U_C}{2}\sum_{t=0}^{\infty}\beta^t[a_1(y_t)^2 + a_2\pi_t^2 + a_3(c_t^B - c_t^R)^2 + t.i.p. + \mathcal{O}(3)]. \tag{A.67}$$

where $a_2 = \frac{\eta}{\delta}$ and $a_3 = \alpha(1-\alpha)[(1-\bar{g})\sigma + \frac{\sigma^2}{\gamma^2}]$. Taking unconditional expectations yields

$$\mathcal{W} \equiv -\frac{\bar{Y}u_c}{2(1-\beta)}[a_1 var(y_t) + a_2 var(\pi_t) + a_3 var(c_t^B - c_t^R) + t.i.p. + \mathcal{O}(3)]. \tag{A.68}$$

Hence, the approximated utility loss is given by

$$L \simeq a_1 var(y_t) + a_2 var(\pi_t) + a_3 var(c_t^B - c_t^R). \tag{A.69}$$

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